Long-range and selective coupler for superconducting flux qubits

Hayato Nakano, Kosuke Kakuyanagi,

NTT Basic Research Laboratories, NTT Corporation, Atsugi-shi, Kanagawa 243-0198, Japan

Masahito Ueda

Department of Physics, Tokyo Institute of Technology, Meguro-ku, Tokyo 152-8551, Japan

NTT Basic Research Laboratories, NTT Corporation, Atsugi-shi, Kanagawa 243-0198, Japan

and Kouichi Semba

NTT Basic Research Laboratories, NTT Corporation, Atsugi-shi, Kanagawa 243-0198, Japan

Abstract

We propose a qubit-qubit coupling scheme for superconducting flux quantum bits (qubits), where a quantized Josephson junction resonator and microwave irradiation are utilized. The junction is used as a tunable inductance controlled by changing the bias current flowing through the junction, and thus the circuit works as a tunable resonator. This enables us to make any qubits interact with the resonator. Entanglement between two of many qubits whose level splittings satisfy some conditions, is formed by microwave irradiation causing a two-photon Rabi oscillation.Since the size of the resonator can be as large as sub-millimeters and qubits interact with it via mutual inductance, our scheme makes it possible to construct a quantum gate involving remote qubits.

One of the major challenges in solid-state quantum computation is to selectively couple two among many qubits that are located at different places in the system. In this letter, we present a scheme to overcome this difficulty.

A practical qubit-qubit coupler should allow, (i) coupling between qubits can be switched on/off at will; (ii) coupling that is strong enough so that the desired entanglement can be formed within the coherence time of the qubit system, (iii) selective coupling between two arbitrary qubits among many qubits, (iv) coupling between two remote qubits.

For superconducting qubits, various coupling schemes have been proposed. Some of them add the third element [1–6] to achieve controllability in two-qubit coupling.

Schematical original idea has appear in Ref. 1. Bias current control of the coupling was shown in Ref. 2. You *et al.* [3] proposed a two-qubit gate with a superconducting LC circuit. Its mechanism is an analogue of the Cirac-Zoller scheme originally proposed for atomic or ionic qubit systems [7]. A DC-SQUID is used as the medium for the interaction in the proposal by Plourde [4] where the SQUID is used as a flux-transformer controlled by changing an external magnetic field and applied DC bias current. Bertet *et al.* [5] improved the controllability of the SQUID flux-transformer by introducing AC bias current into the SQUID.A third qubit is introduced as the medium of two qubits in the proposal by Niskanen et al. [6].

Above schemes [3–6] satisfy the requirments (i) and (ii). One [3] can be used a qubit-qubit coupler allowing us to couple spacially separated qubits, that is, to satisfy requirement (iv). In particular, requirement (iv) is important when we integrate many flux-qubits on the same chip using microfabrication techniques, in order to directly couple two target qubits that do not neighbor each other.

The qubit-qubit coupling scheme we proposed here for superconducting flux qubits satisfies all of the above requirements. We use a tunable resonator and two-photon Rabi oscillation. The resonant frequency of a circuit consisting of capacitance C and inductance L is given by

$$\omega_{\rm r} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(L_0 + L_{\rm J})C}}, \tag{1}$$

where we put $\hbar = 1$. L_0 is the geometrical and kinetic inductance of the circuit, and $L_{\rm J}$ stands for the Josephson inductance, $L_{\rm J} = (\frac{1}{2e})^2/E_{\rm J}$ where $E_{\rm J}$ is the Josephson energy of the junction. The effective Josephson energy, i.e., the curvature at the bottom of the Josephson potential varies with the DC bias $I_{\rm b}$ flowing through the junction as

$$E_{\rm J} = \sqrt{{E_{\rm J0}}^2 - (I_{\rm b}/(2e))^2} - {\gamma'}^2 I_{\rm b}/(2e), \tag{2}$$

where E_{J0} is the Josephson energy in the absence of a bias current, and γ' is the Josephson phase measured from the potential bottom position. The potential barrier against the switching of the junction is approximately given by $\Delta V \simeq \frac{3}{\sqrt{2}} E_{\mathrm{J0}} x^{3/2}(1 - x/24)$, where $x = I_{\mathrm{b}}/(2eE_{\mathrm{J0}})$. Therefore, we can approximately regard the circuit as a tunable LC resonator (a harmonic oscillator) under the conditions $\gamma'^2 \ll 2eE_{\mathrm{J}}/I_{\mathrm{b}}$ and $k_{\mathrm{B}}T, E_{\mathrm{noise}} \ll \Delta V$. The former condition is necessary in order to neglect the nonlinearity of the potential and it restricts the amplitude of the resonator. The latter prevents unwanted excitations, *e.g.*, those due to thermal, noise, from causing switching. As a result, by changing the bias current, we can tune the resonant frequency of a Josephson junction resonator circuit from 70 % to 100 % of that in an unbiased resonator.

Suppose that there are a number of flux qubits ($i = 1, 2, \ldots$) in a region of sub-millimeter size. The level splitting of each qubit is $\omega_i = \sqrt{{\varepsilon_i}^2 + {\Delta_i}^2}$, where ε_i can be controlled by changing the external magnetic flux piercing the qubit ring, and Δ_i is the tunneling matrix element of the qubit. In superconducting flux qubit systems, level splitting ω_i between two states is usually different for each qubit. All qubits interact with the same Josephson resonator discussed above via mutual inductance M_i ($i = 1, 2, \ldots$), as illustrated in Fig. 1. When we represent the qubits with Pauli matrices and the resonator with creation and annihilation operators, we obtain the Hamiltonian

$$\begin{aligned} H = \sum_i \frac{\omega_i}{2}\sigma_{iz} + \omega_a \left(a^\dagger a + \frac{1}{2}\right) \\ + \sum_i g_i (a^\dagger + a)(\cos\frac{\theta_i}{2}\sigma_{iz} - \sin\frac{\theta_i}{2}\sigma_{ix}) \\ - \sum_i (\cos\frac{\theta_i}{2}\sigma_{iz} - \sin\frac{\theta_i}{2}\sigma_{ix}) f_i \cos\omega_{\mathrm{ex}} t, \end{aligned} \qquad (3)$$

where $\tan\theta_i = \Delta_i/\varepsilon_i$, and $g_i \propto M_i$ is the qubit-resonator coupling constant. Here, the last term corresponds to microwave irradiation to qubits with the frequency ω_{ex}.

Now we choose two qubits $i = 1, 2$ and consider the coupling between them. First, we tune the frequency of the resonator to $\omega_{\mathrm{r}} = \frac{1}{2}(\omega_1 + \omega_2) + \delta$. Then, we apply a microwave of the frequency $\omega_{\mathrm{ex}} = \frac{1}{2}(\omega_2 - \omega_1) + \delta_{\mathrm{ex}}$.

The energy diagram of the two-photon Rabi oscillation caused by the microwave irradiation is illustrated in Fig. 2 [8]. Here, the notation of the state, $|g,\ e,\ n\rangle$ means that qubit 1 is in its ground state and qubit 2 is in its excited state, and the resonator is in the boson number state of $|n\rangle$. When we expect the transition between $|e,\ g,\ n\rangle$ and $|g,\ e,\ n\rangle$, the intermediate state is $|g,\ g,\ n+1\rangle$ or $|e,\ e,\ n-1\rangle$. When the detuning δ is exactly equal to 0, the energies of these intermediate states are placed exactly at the middle between $|g,\ e,\ n\rangle$ and $|g,\ e,\ n\rangle$. Then, the irradiation resonates to

the single photon transition to/from the intermediate states and real transitions occur. By applying a finite detuning δ, such real transitions are suppressed and the two-photon transitions between $|e,\ g,\ n\rangle$ and $|g,\ e,\ n\rangle$ are induced. In the absence of the resonator, the transition is negligibly small.

Next, we show the results of numerical calculations. For simplicity, parameter values were fixed as follows: $\omega_1 = 1.0$, $\omega_2 = 1.2$, $\theta_1 = \theta_2 = \pi/6$, $g_1 = 0.05$, $g_2 = 0.06$, $f_1 = 0.02$ and $f_2 = 0.03$. Moreover, the irradiation frequency was set as $\omega_{\rm ex} = \frac{1}{2}(\omega_2 - \omega_1) + \delta_{\rm ex}$. The detuning $\delta_{\rm ex} = 0.0074$ was applied in order to resonate the true energy splitting affected by the coupling with the resonator. The resonator frequency $\omega_{\rm r} = \frac{1}{2}(\omega_1 + \omega_2) + \delta$ with δ from -0.1 to +0.1 was examined. The time evolution of the density operator of the two-qubit-resonator composite system were calculated by solving the master equation $\frac{\rm d}{{\rm d}t}\rho = \frac{1}{i}[H, \rho] + \frac{\Gamma}{2}(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a)$ with the backward-Euler method. This master equation corresponds to the case where linear loss in the resonator is considered.

Figure 3 shows the time evolution of the system without any decoherence, i.e., $\Gamma = 0$. The detuning of the resonator is $\delta = -0.02$. The initial state is $|e, g, 0\rangle$. In Fig. 3(a), we can see the population of the initial state is transfered to the $|g, e, 0\rangle$. This is the Rabi oscillation of $|e,\ g,\ 0\rangle \leftrightarrow |g,\ e,\ 0\rangle$, which forms an entangled state $|\psi\rangle = \alpha|e,\ g,\ 0\rangle + e^{i\varphi}\sqrt{1-|\alpha|^2}|g,\ e,\ 0\rangle$.

To confirm the entanglement, we show the time evolution of the concurrence C of the qubits in Fig. 3(b). The concurrence is a measure of the extent of entanglement and defined by $C = \sqrt{\lambda_1} - \sum_{k>1}\sqrt{\lambda_k}$, where $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \ldots$, and λ_k is an eigenvalue of $\rho_y\rho_y^*$ with the density operator $\rho_y = \sigma_{1y}\sigma_{2y}\rho$.

$C = 1$ means that a maximally entangled qubit pair is formed. In our calculation, C approaches unity, although a rapid oscillation is superposed. The rapid oscillation comes from the real excitation of the state $|g,\ g,\ 1\rangle$. This type of real excitation is much suppressed when the microwave irradiation is weak. However, under weak irraditaion, the period of the two-photon Rabi oscillation becomes long resulting in taking a long time for entangling two qubits. As discussed below, entanglement formation should be completed within the coherence time. Therefore, weak irraditation is not appropriate. Some optimization, such as detuning of the resomator frequency is still possible to suppress unwanted real excitation.

We also examined the influence of decoherence on the behavior of the coupler.

An almost maximal entanglement can be formed when we use a resonator with the Q value of more than one thousand. The formation takes a few hundred times the period of the Larmor rotations of qubits, $2\pi/\omega_i$. For a qubit of $\omega_i \sim 4$ GHz, this approximately corresponds to 100

nanoseconds. Therefore, a controlled-NOT gate containing approximately ten two-qubit operations, can be constructed when the coherence time of qubits exceeds microseconds. This condition can be met in the near future.

It is worth mentioning that this coupler can make an interaction between remote qubits because they interact with the same resonator whose size can become the order of sub-millimeter. This makes the entanglement formation between remote qubits considerably fast compared to that using successive nearest neighbor interactions.

The resonant conditions are rather strict. When the frequencies deviate by a few percent from the conditions, two-photon transition becomes negligibly small. This strictness offers a good selectivity of the target pair of qubits from other qubits. We can switch the target qubit-pair within a few nanoseconds by changing the DC-bias and the irradiation frequency.

In summary, we proposed a qubit-qubit coupler for superconducting flux qubits, using a Josephson resonator and a microwave irradiation. The coupler can make an interaction between two remote qubits among many qubits. We have already performed a quantum interaction behavior of a composite system consisting of a flux-qubit and a superconducting LC resonator [9]. Moreover, we have succeeded in controlling two-photon Rabi oscillations in single flux-qubit systems [10]. Therefore, we have the elementary experimental techniques necessary to realize the coupler discussed here.

This work was partially supported by CREST-JST.

References

[1]Y. Makhlin, A. Shnirman and G. Schön, Rev. Mod. Phys., **73**, 357 (2001).

[2] A. Blais, A. Massan van den Brink and A. M. Zagoskin, Phys. Rev. Lett., **90**, 127901 (2003).

[3] J. Q. You, J. S. Tsai, and Franco Nori, Phys. Rev. **B 68**, 024510 (2003).

[4] B. L. T. Plourde, J. Zhang, K. B. Whaley, F. K. Wilhelm, T. L. Robertson, T. Hime, S. Linzen, P. A. Reichardt, C.-E. Wu, and John Clarke, Physi. Rev. **B 70**, 140501(R) (2004).

[5] P. Bertet, C. J. P. M. Harmans, and J. E. Mooij, Phys. Rev. **B 73** , 064512 (2006).

[6] A. O. Niskanen, K. Harrabi, F. Yoshihara, Y. Nakamura, and J. S. Tsai, Phys. Rev. **B 73**, 094506 (2006).

[7] J. I. Cirac and P. Zoller, Phys. Rev. Lett., **74**, 4091 (1995).

[8] A qubit-qubit coupler for a trapped ion system using similar two-photon Rabi oscillation was proposed in, A. Sørensen and K. Mølmer, Phys. Rev. Lett. **82**, 1971 (1998). However, their resonator is not tunable and the energy diagram is different.

[9] J. Johansson, S. Saito, T. Meno, H. Nakano, M. Ueda, K. Semba, and H. Takayanagi, Phys. Rev. Lett. **96**, 127006 (2006).

[10] S. Saito, T. Meno, M. Ueda, H. Tanaka, K. Semba, and H. Takayanagi, Phys. Rev. Lett. **96**, 107001 (2006).

Figure Captions

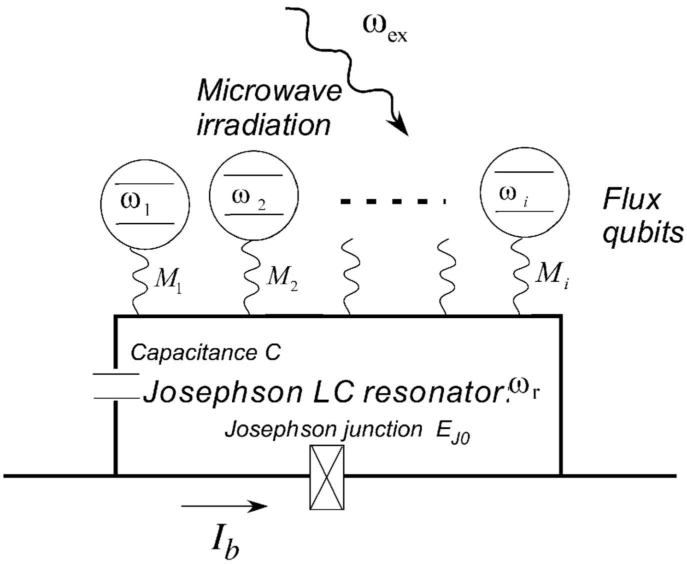

Figure 1:

Flux qubits interacting with a common Josephson resonator. The resonance frequency of the resonator can be tuned by changing the bias current $I_{\rm b}$. Each flux qubit with resonant frequency ω_i couples to the resonator via mutual inductance M_i. We apply a microwave of frequency $\omega_{\rm ex} \simeq (\omega_i \pm \omega_j)$ to make entanglement between the qubits i and j.

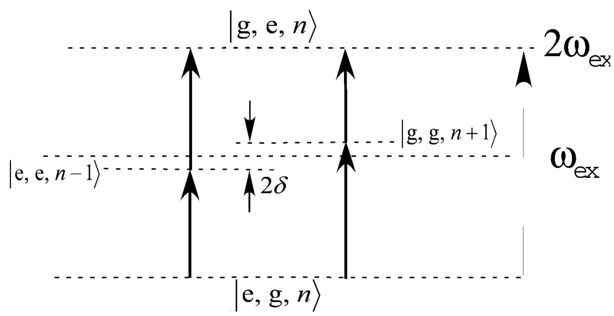

Figure 2:

Schematic energy diagram for the two-photon Rabi oscillation in the two-qubit-resonator system. The Rabi oscillation causes coherent transition between $|e, g, n\rangle$ and $|g, e, n\rangle$. The energy splitting between these two states corresponds to two times the microwave frequency $\omega_{\rm ex}$. This is a nonlinear (second-order) transition. Energies of the intermediate states $|e, e, n-1\rangle$ and $|g, g, n+1\rangle$ are shifted to approximately the middle between $|e, g, n\rangle$ and $|g, e, n\rangle$ by the interaction with the resonator, resulting in enhancement of the nonlinear (second-order) transition.

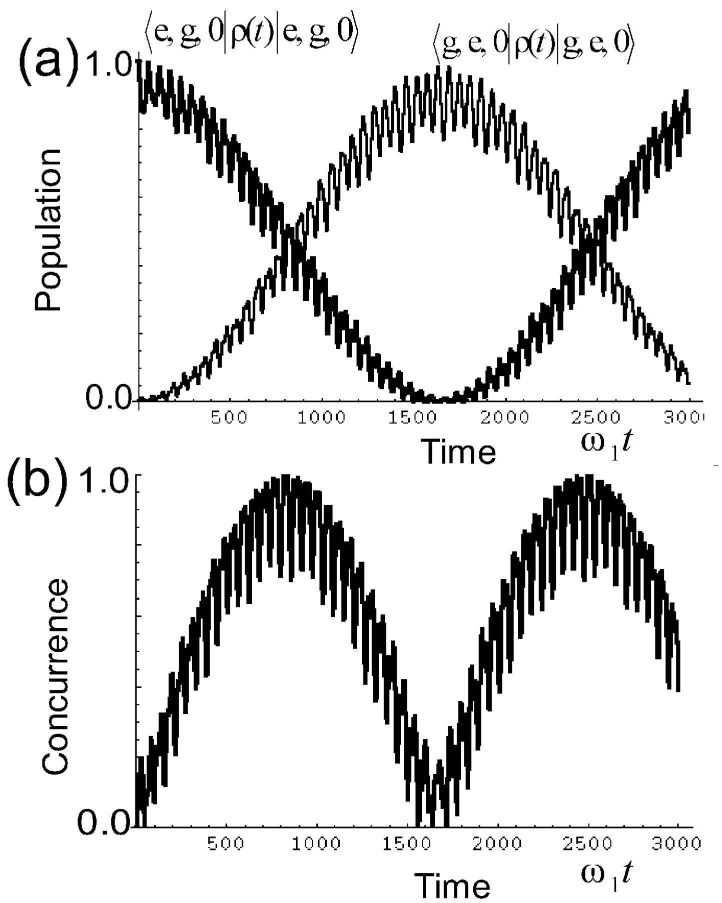

Figure 3:

Time evolution caused by microwave irradiation, where, there is no decoherence in the system. The initial state is $|e, g, 0\rangle$. Rabi oscillation makes a superpostion of $|e, g, 0\rangle$ and $|g, e, 0\rangle$, which is an entangled state. (a) populations of the two states. (b) concurrence of the entangled state.

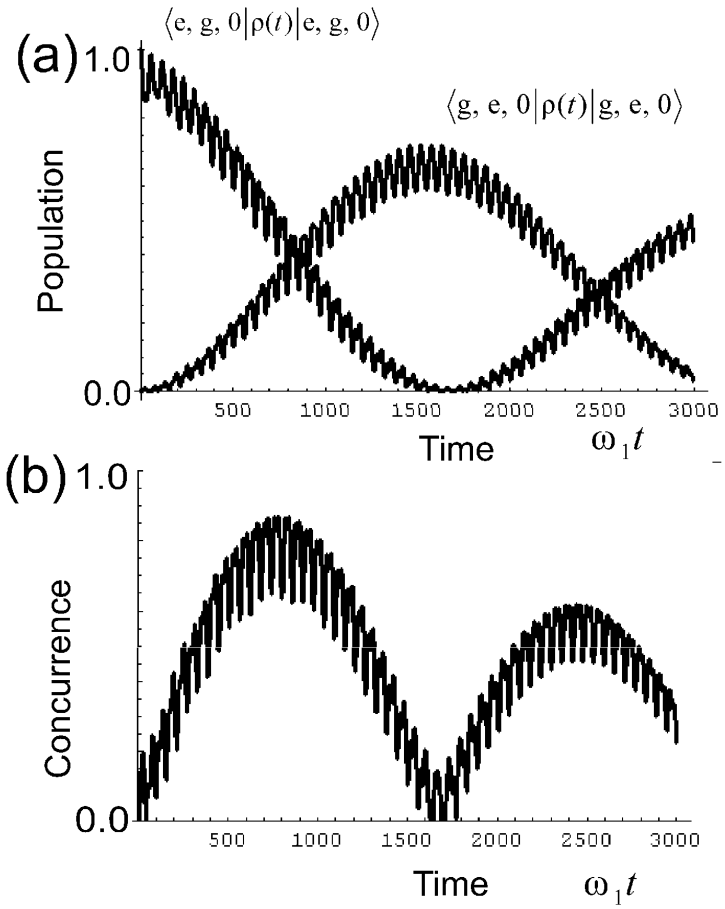

Figure 4:

Time evolution caused by maicrowave irradiation, with decohererence ($\Gamma = 0.002$). Linear loss in the resonator is provided. There is no direct decoherence to qubits. (a) populations in the two states. (b) concurrence of entanglement. Entanglement is much poluted by decoherence.